**PubHtlh 540 Introductory Biostatistics**
# Review of Logarithms and Exponents

## An example of logarithms in everyday life

- In the Richter scale for earthquakes, an increase of one point on the scale corresponds to an increase of 10 times more energy released.. The Richter scale for earthquakes is a base 10 logarithmic scale

## Logarithms and Exponents are Useful in Biostatistics

- When a distribution is skewed (as for example when it has a long right tail), it is sometimes hard to see the structure in the data. In such instances, taking logarithms of the data and visualizing the data on the logarithmic scale will make it much easier to identify structures in the data.

- Sometimes, the mathematical calculations required in an analysis of data are easier to do using techniques of logarithms and exponents.

- Maximum likelihood estimation (not covered in this course) utilizes the natural logarithm of likelihood functions.

## Introduction to the language and structure of logarithms and antilogarithms

- The word **"logarithm"** is just another word for "exponent" or "power". For example, in the expression $3^4 = 81$, the digit "4" can be called a logarithm.

- The word "**antilogarithm**" is just another word for "number" or "result". For example, in the expression $3^4 = 81$, the result "81" is the antilogarithm.

**Logarithm (exponent, power)**

$$3^4 = 81$$

Base

Number (result, **antilogarithm**)

- We live in the world of **base 10.** We work with 10 digits (they are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9). Thus, when we count, we count 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and we want to increment again, the next count has to be 10.

- Computers live in the world of **base 2**. They work with two digits (they are 0 and 1). When they count, they count 0, 1 and when they want to increment again, the next count has to be 10. So here's how it proceeds.

| Number in Base 10 | Its representation in Base 2 |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 10 |
| 3 | 11 |
| 4 | 100 |
| 5 | 101 |
| 6 | 110 |
| 7 | 111 |
| 8 | 1000 |
| 9 | 1001 |

**Some Examples**

| | | Base | Logarithm (exponent) | Number (antilogarithm) |
|---|---|---|---|---|
| $2^3 = 8$ | $\log_2 [8] = 3$ | 2 | 3 | 8 |
| $7^2 = 49$ | $\log_7 [49] = 2$ | 7 | 2 | 49 |
| $b^0 = 1$ | $\log_b [1] = 0$ | b | 0 | 1 |

**Laws of Logarithms**

1. $\log_b(MN) = \log_b(M)+\log_b(N)$

2. $\log_b(M/N) = \log_b(M)-\log_b(N)$

3. $\log_b(M^p) = (p)\log_b(M)$

4. **Common** logarithms are those for which the base b = 10. There are two notations:

   $\log_{10}[\ \ ]$ ← ***Hint: This is the better notation to use.***
   $\log[\ \ ]$

5. **Natural** logarithms are those for which the base b = e = 2.71828. There are two notations:

   $\log_e[\ \ ]$
   $\ln[\ \ ]$ ← ***Hint: This is the better notation to use.***

**Laws of Exponents**

1. $\left[a^p\right]\left[a^q\right]=\left[a^{p+q}\right]$

2. $a^p/a^q = a^{p-q}$

3. $\left(a^p\right)^q = a^{pq}$

4. $a^0 = 1$ provided a is not zero.

5. $a^{-p} = 1/a^p$

6. $(ab)^p = \left(a^p\right)\left(b^p\right)$

## Worked Examples

1. **Question: Find the common logarithm (base 10) of 36. Call it x.**
   **Answer: x = 1.5563**

   Solution:
   "common logarithm (base 10) of 36. Call it x". →
   $\log_{10}[36] = x$ → $10^x = 36$
   From a table of logarithms, or using a calculator, obtain x=1.5563

2. **Question: In base 10, what is the antilogarithm of 0.91166? Call this x.**
   **Answer: x = 8.1651**

   Solution:
   "antilogarithm of 0.91166" → $\text{antilog}_{10}$ (0.91196) = x →
   $10^{0.91196} = x$
   Using a calculator, obtain x=8.1651